

Problems

Ted Eisenberg, Section Editor

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <<http://www.ssma.org/publications>>.

*Solutions to the problems stated in this issue should be posted before
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5529: *Proposed by Kenneth Korbin, New York, NY*

Convex cyclic quadrilateral $ABCD$ has integer length sides and integer area. The distance from the incenter to the circumcenter is 91. Find the length of the sides.

5530: *Proposed by Arsalan Wares, Valdosta State University, Valdosta, GA*

Polygon $ABCD$ is an 11 by 12 rectangle ($AB > AD$). Points P, Q, R , and S are on sides AB, BC, CD , and DA , respectively, such that PR and SQ are parallel to AD and AB , respectively. Moreover, $X = PR \cap QS$. If the perimeter of rectangle $PBQX$ is $5/7$ the perimeter of rectangle $SAPX$, and the perimeter of rectangle $RCQX$ is $9/10$ the perimeter of rectangle $PBQX$, find the area of rectangle $SDRX$.

5531: *Proposed by Daniel Sitaru, National Economic College "Theodor Costescu," Drobeta Turnu-Severin, Mehedinti, Romania*

For real numbers x, y, z prove that if $x, y, z > 1$ and $xyz = 2\sqrt{2}$, then

$$x^y + y^z + z^x + y^x + z^y + x^z > 9.$$

5532: *Proposed by Arkady Alt, San Jose, CA*

Let a, b, c be positive real numbers and let $a_n = \frac{an + b}{an + c}, n \in N$. For any natural number

$$m \text{ find } \lim_{n \rightarrow \infty} \prod_{k=n}^{nm} a_k.$$

5533: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Find the value of the sum

$$\sum_{n=1}^{+\infty} \frac{n^2 \alpha^n}{(n-1)!}$$

for any real number $\alpha > 0$. (Here, $0! = 1! = 1$).